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Ultimate stable element Z = 137

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Abstract: We have analysed and highlighted an interesting prediction of Subrahmanyam Chandrasekhar that Z = 137 will be the ultimate element that can be artificially synthesised. The heaviest element found till today is of atomic number Z = 122 and atomic mass A = 292. The last possible stable element has, in fact, been a matter of disagreement.

Keywords: Heavy nuclei, Bohr's model, special theory of relativity, Dirac equation.

Introduction

The discovery of new elements has been a topic of considerable interest for more than half a century. We have only 92 naturally occurring elements on the earth. Hydrogen, the lightest element, has one proton in its nucleus and uranium the heaviest naturally occurring element has ninety two protons. Towards the end of 1945 all these ninety two elements were filled in Mendeleev's periodic table. The first artificial radioactive element with atomic number (Z=93) was discovered by Edwin McMillan and Philip H. Abelson in the year 1940 in Berkeley, California. It was named Neptunium. Since then twenty nine more artificial elements have been synthesised either by the process of nuclear fission or particle acceleration. Though often short-lived these artificial elements provide scientists with valuable insight into the structure of atomic nuclei. It also offers opportunities to study the chemical properties of the heavier elements beyond Uranium.

Recently (Marinov *et al.*, 2007,2008; Brumfiel, 2008) it is claimed that the element with atomic mass A = 292 and atomic number Z = 122 (eka-Th) as the heaviest element (Sahoo, 2008). Prof. Amnon Marinov of the Hebrew University in Jerusalem and their group took a purified sample of thorium and used an electric field to accelerate the nuclei. Then they passed them through a magnet, whose field bent lighter nuclei more than heavier ones. Using plasma-sector field mass spectroscopy they separate the heaviest nuclei. Their results show the existence of a superheavy nucleus with atomic number Z = 122, atomic mass A = 292 and abundance $(1-10)\times 10^{-12}$ relative to ^{232}Th. Its half-life $t_{1/2} \geq 10^8$ y suggests that it is a long-lived isomeric state exists in this isotope. But it is not confirmed fully. According to Prof. Rolf-Dietmar Herzberg, a nuclear physicist at the University of Liverpool, UK, more evidences are required.

In recent years there seems to be a growing realization that the next neutron magic number is 184 (Kumar, 1989). The next proton (Z) magic number is yet a matter of disagreement. However, it can be recalled here that quite sometime back in 1982 famous astrophysicist Sir Chandrasekhar had given a hint on the limit of stable elements. His predicted theoretical proton limit is given by the reciprocal of the so called fine structure constant, namely $\frac{\hbar c}{e^2} \approx 137$. But that is still out of reach of the current experiment. If it comes true then element with Z = 137 will be the last of the island of stability that can be artificially synthesized in the laboratory. In the next section using Bohr's model and special theory of relativity we have mathematically illustrated Chandrasekhar's reason that one can have no stable atomic structure with a nuclear charge in excess of Z = 137.

Nuclear charge and orbital velocity of electron

As per Bohr's model the first Bohr orbit of hydrogen atom (Z = 1) is given by $a_0 = \frac{\hbar^2}{m_e e^2}$ (Beiser, 1997; Cohen, 1999), where the lone electron revolves around the nucleus in an orbit of radius 'a_0'. Now consider a singly charged helium atom i.e. when one electron from the orbit of ^4_2He is removed we get $^4_2He^+$ atom. Let us now compare the radius of a singly charged helium atom with that of a hydrogen atom.

In case of a singly charged helium atom the electrostatic attraction between nucleus (+ e) and electron (- e) is $-\frac{e^2}{r^2}$. Therefore, the potential energy of the above atom is given by $E_P = -\frac{e^2}{r}$. (1)

We know that in atoms there exists preferred orbits and de Broglie waves are wrapped around each orbit. However the wavelengths are different for different orbits (Chandrasekhar, 1984; Venkataraman, 1993, 2002). The wavelength of a particular orbit is determined by the kinetic energy of revolving electron. Whereas the radius of a particular orbit is determined by the balance between the potential and kinetic energy. The kinetic energy of the electron of mass m_e revolving around the nucleus with velocity v is given by the relation

$$E_K = \frac{1}{2}m_e v^2 = \frac{p^2}{2m_e}, \quad (2)$$

where $p = m_e v$ is the momentum and it is related to λ by $\lambda = \frac{h}{p}$, where h is the Planck's constant.

In case of singly charged helium atom the lone electron in the orbit interacts with two protons of the

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nucleus. So, two waves are wrapped around the same orbit. So, each wave contributes $\lambda/2$ to the orbit of radius r. Hence we get

$$\lambda/2 = 2\pi r \quad \text{or} \quad \lambda = 4\pi r. \tag{3}$$

Using the relation $\lambda = \frac{h}{p}$ we get from Eq. (3) the expression for the momentum of the electron as

$$p = \frac{\hbar}{2r}. \tag{4}$$

Therefore the kinetic energy of the electron is given by

$$E_K = \frac{p^2}{2m_e} = \left(\frac{\hbar^2}{4m_e}\right).\frac{1}{2r^2}. \tag{5}$$

So, the total energy E of the electron is

$$E = -\frac{e^2}{r} + \left(\frac{\hbar^2}{4m_e}\right).\frac{1}{2r^2} = -\frac{C}{r} + \frac{B}{2r^2}, \tag{6}$$

where $C = e^2$ and $B = \frac{\hbar^2}{4m_e}$. Above expression implies that the electron tries to maintain a compromise between the electrostatic energy and the kinetic energy. The balance is kept to a minimum. Taking the derivative of Eq. (6) we find

$$\frac{dE}{dr} = \frac{C}{r^2} - \frac{B}{r^3}. \tag{7}$$

To find the value of r when E is minimum we must set $\frac{dE}{dr} = 0$ and solve it for r. we then get

$$r = \frac{B}{C} = \frac{1}{4}\left(\frac{\hbar^2}{m_e e^2}\right) = \frac{1}{4}a_0. \tag{8}$$

This is the effective value of r where the total energy is minimum. We thus find here that the radius of a singly charged helium atom is one-fourth of the radius of hydrogen atom. When an electron in a hydrogen atom circulate around the nucleus in the orbit given by radius a_0, with a velocity v then from the relation equating the attractive electrostatic force to the centrifugal force we get

$$\frac{m_e v^2}{a_0} = \frac{e^2}{a_0^2} \tag{9}$$

Or $v = e\sqrt{1/m_e a_0}$, (10)

is the velocity of electron in a hydrogen atom (Z = 1). Now, in case of singly charged helium atom (Z = 2) as the electron circulate in the orbit of radius $a_0/4$, its velocity v' becomes $v' = 2e\sqrt{1/m_e a_0} = 2v$. (11)

We thus find that in singly charged helium atom the electron circulate around the nucleus with a velocity twice as large as in case of hydrogen atom. When we analyse Eq. (10) and Eq. (11), we see that as the nuclear charge increases from Z = 1 to Z = 2, the electron orbits in hydrogen like atom come closer to the centre and velocity of orbital electron increases. But there must be a maximum velocity of an orbital electron for a stable element. The ultimate limit of orbital velocity will be attained when it approaches the velocity of light. As we know according to special theory of relativity, no particle can have a velocity exceeding that of light. Moreover, when a particle moves with a velocity close to that of light its effective mass increases. By including such limitations as prescribed by special theory of relativity (STR) we can show that there exist a maximum charge (Z) limit for which the orbital velocity of electron can have a ultimate velocity ($v = c$) to provide stability to the atom.

Maximum nuclear charge for stability

Consider a hydrogen like atom with nuclear charge 'Ze' and the electron is moving round the nucleus in an orbit of radius 'r'. Equating the attractive electrostatic force to the centrifugal force for the stability of the atom, we have

$$\frac{Ze^2}{r^2} = \frac{m_e v^2}{r}. \tag{12}$$

If the orbital electron moves with a velocity v, the mass of electron becomes

$$m_e = \frac{m_0}{\sqrt{1 - v^2/c^2}}. \tag{13}$$

Now from Eq. (12) and Eq. (13) we get

$$Z = \frac{m_e v^2 r}{e^2} = \frac{m_0 v^2 r}{\left(\sqrt{1 - v^2/c^2}\right).e^2} \tag{14}$$

In order to get a maximum charge (Z) limit, the velocity of orbital electron should be the maximum velocity which is equal to c. Now when $v \to c$, the effective mass of the electron increases indefinitely [Eq. (13)] and Z go off to infinity [Eq. (14)]. Hence, in this case this semi-classical approach must be modified to take into account the effects predicted by the theory of special relativity. The corresponding motion of the electron is defined by Dirac equation (Dirac, 1928; Dutt & Ray 1993; Sakurai, 2003) and its solution. The Dirac equation in the Hamiltonian form can be written as:

$$H\psi = i\hbar\frac{\partial\psi}{\partial t}, \tag{15}$$

where

$$H = -ic\hbar\vec{\alpha}.\vec{\nabla} + \beta mc^2, \tag{16}$$

with

$$\beta = \gamma_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \alpha_k = i\gamma_4\gamma_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}, \tag{17}$$

where γ_μ ($\mu = 1, 2, 3, 4$) are 4×4 matrices (known as gamma matrices or Dirac matrices) given by

$$\gamma_k = \begin{pmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad (18)$$

σ_k are the Pauli matrices and given by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad (19)$$

The symbol I stands for the 2×2 identity matrix

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \qquad (20)$$

The gamma matrices satisfy the following anticommutation relations:

$$\left\{\gamma_\mu, \gamma_\nu\right\} = \gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\,\delta_{\mu\nu}. \qquad (21)$$

The solution for the energy levels of the Dirac equation using the Coulomb potential, $V(r) = -\frac{Ze^2}{r}$, can be written as:

$$E_{n,j}^{rel} = m_0 c^2 \left[1 + \frac{(\alpha Z)^2}{\left\{ n - (j+1/2) + \sqrt{(j+1/2)^2 - (\alpha Z)^2} \right\}^2} \right]^{-\frac{1}{2}}, \qquad (22)$$

where m_0 is the electron's rest mass, $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$, $n = j + 1/2 + k = 1, 2......$ the principal quantum number, and $j = 1/2$ for $\ell = 0$ or $j = \ell \pm 1/2$ if $\ell \neq 0$. From equation (22) it is clear that for the smallest value of $j = 1/2$ if $Z > \frac{1}{\alpha}$, the expression under the square root becomes negative and leads to unphysical solutions (Schreiber & Skachkov, 2008; Hotson, 2002).

If we consider the nonrelativistic case, the corresponding formula for the bound state energy levels within the Schroedinger equation can be written as:

$$E_{n,j}^{nonrel} = -\frac{R\hbar Z^2}{n^2}, \qquad (23)$$

where R is the Rydberg constant and $n = 1, 2,........$, is valid for all Z values. In order to resolve the discrepancy between the relativistic solution [Eq. (22)] and nonrelativistic solution [Eq. (23)], the value of Z should not be larger than $1/\alpha = 137$ (Akhiezer & Berestetskii, 1957; Bagrov,1999; Sokolov & Ternov, 1983; Sokolov & Ternov, 1986). Thus, the maximum value of Z = 137.

Here we find that the maximum charge (Z) that an atomic nucleus can have to give stability to the atom is given by the reciprocal of the fine structure constant. The special theory of relativity therefore shows that one can not have a stable atomic structure with a nuclear charge more than Z = 137.

Again a limit on Z is the purview of nuclear forces as described by the semi-empirical mass formula (Liley, 2003) and its famous limit: $\frac{Z^2}{A} \sim 49$. (24)

The Eq. (24) gives the expected atomic mass of the corresponding element A = 383. But its accurate value will be known after its detection in the laboratory.

Conclusion

The synthesis of artificial elements in the laboratory has opened the way for strange new elements that lie beyond uranium. But, very often we wonder that how many such artificial elements can be synthesised in the laboratory? Is there any end to it? Once Nobel Laureate S. Chandrasekhar in his speech had raised such question. He had asked " Why are there just 92 naturally occurring elements? Why are there not a thousand or ten thousand different atomic species". By analysing his answer to this question we see how beautifully he has emphasized a good reasoning based on simple arguments and fundamental constants. We realized that the ultimate limit to the number of elements on earth has been imposed by the ultimate velocity (*v = c*), that a material particle can attain. Special theory of relativity is therefore the grammar of physics that decides many such limits. Our analysis shows an amazing relationship between the ultimate possible element 137 and the fine structure constant (reciprocal relation), a wonderful fundamental constant of nature. The experimental verification of the above theoretical prediction would be very desirable.

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